Multi parton interactions in high energy hadronic collisions

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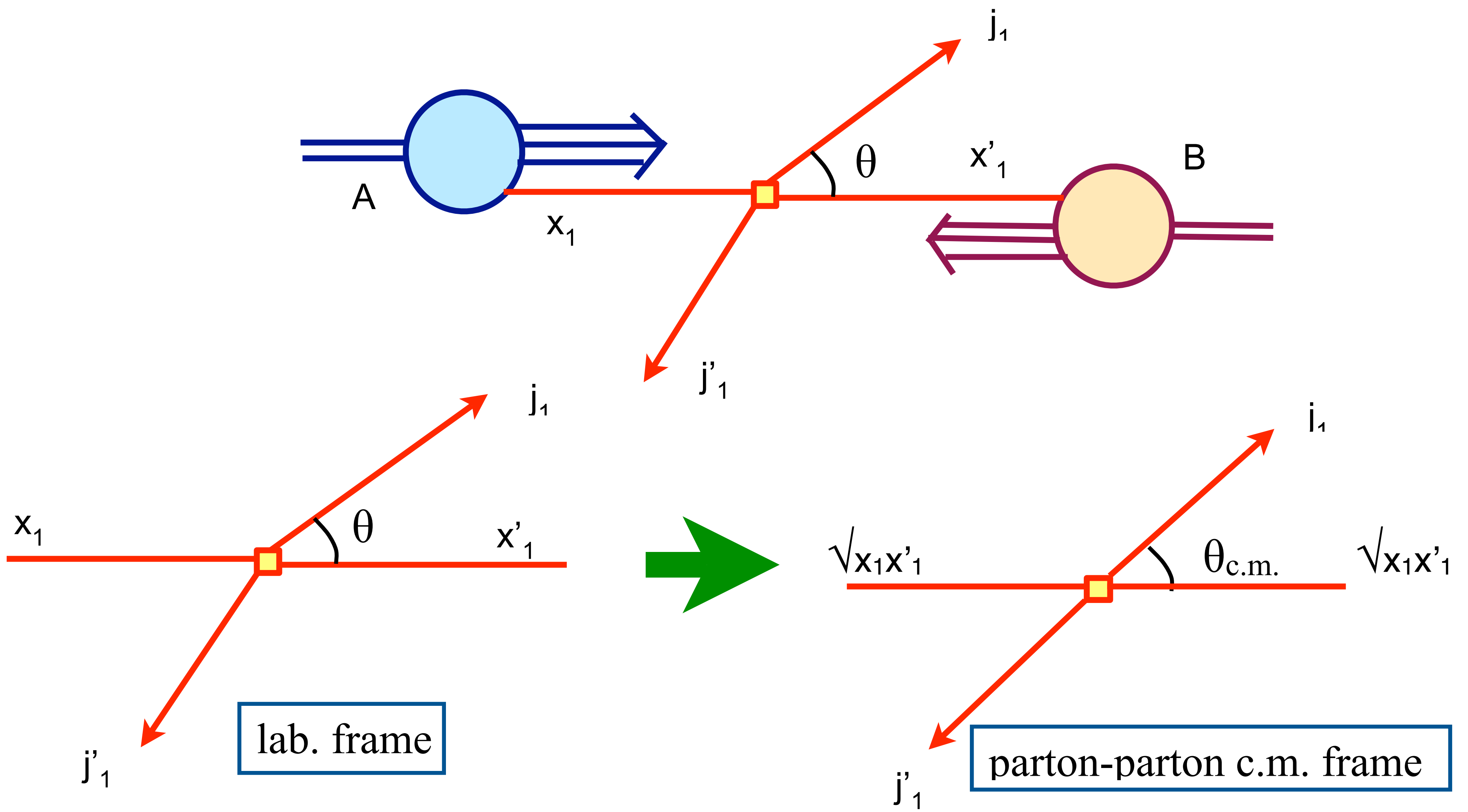
- The UA1 minijets

- Unitarity and the simplest Poissonian model

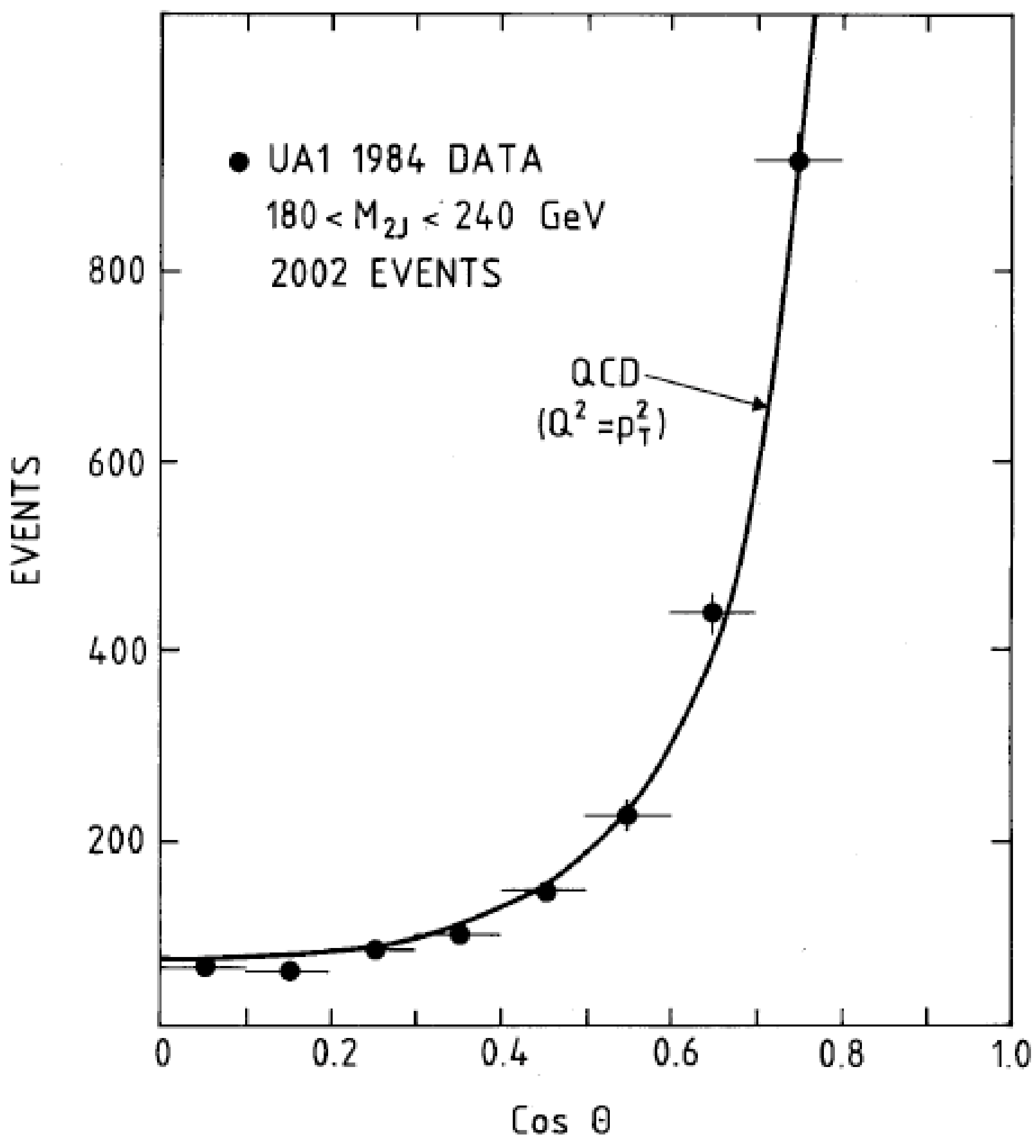
- The CDF analysis of double parton scatterings

- Hadronic fluctuations and multiple parton interactions

Large p_t jet production

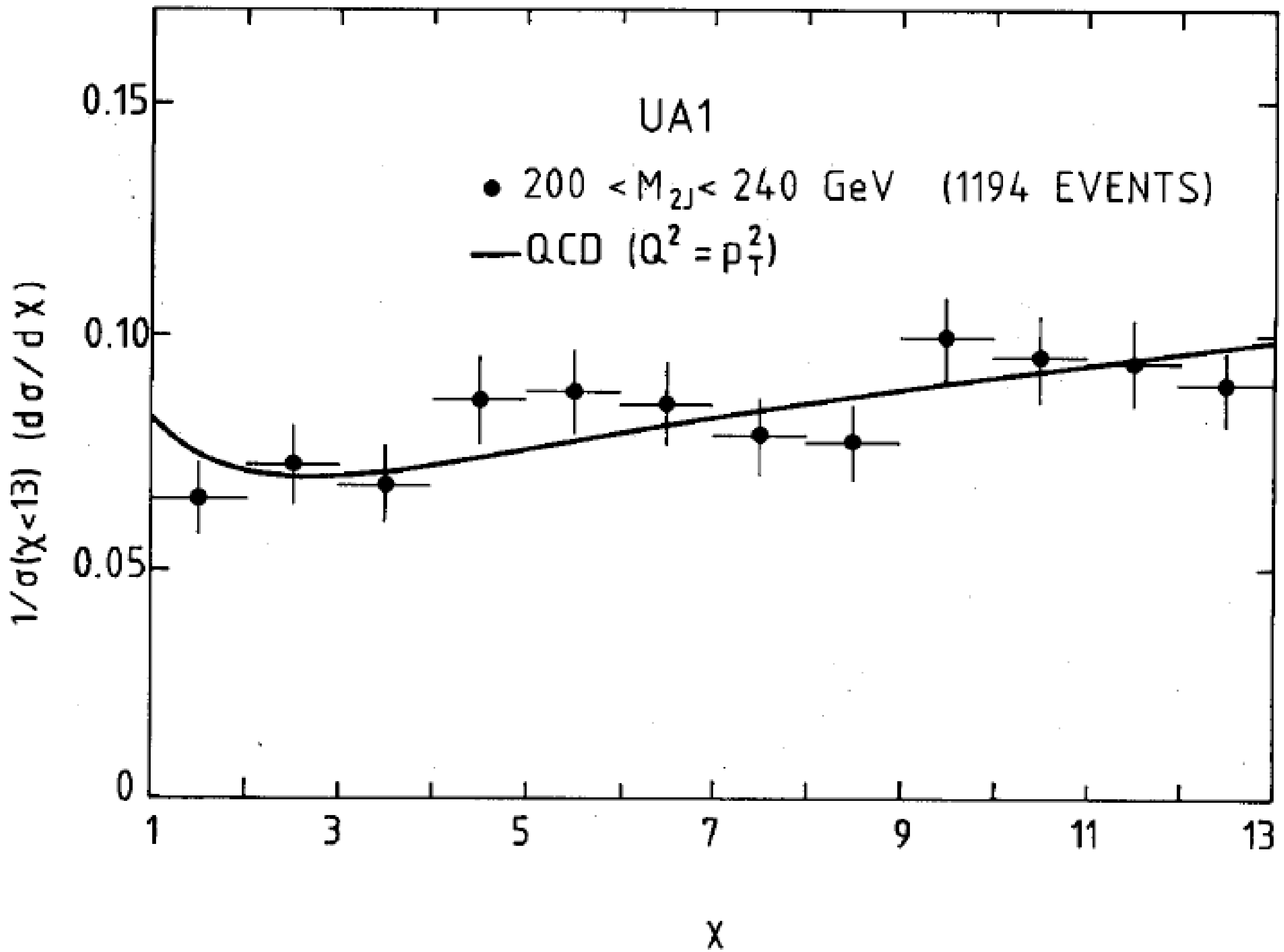


Rutherford singularity



$$\frac{d\hat{\sigma}}{d\Omega} \propto \frac{1}{\sin(\theta_{c.m.}/2)^4}$$

$$\chi \equiv (1 + \cos\theta)/(1 - \cos\theta) .$$



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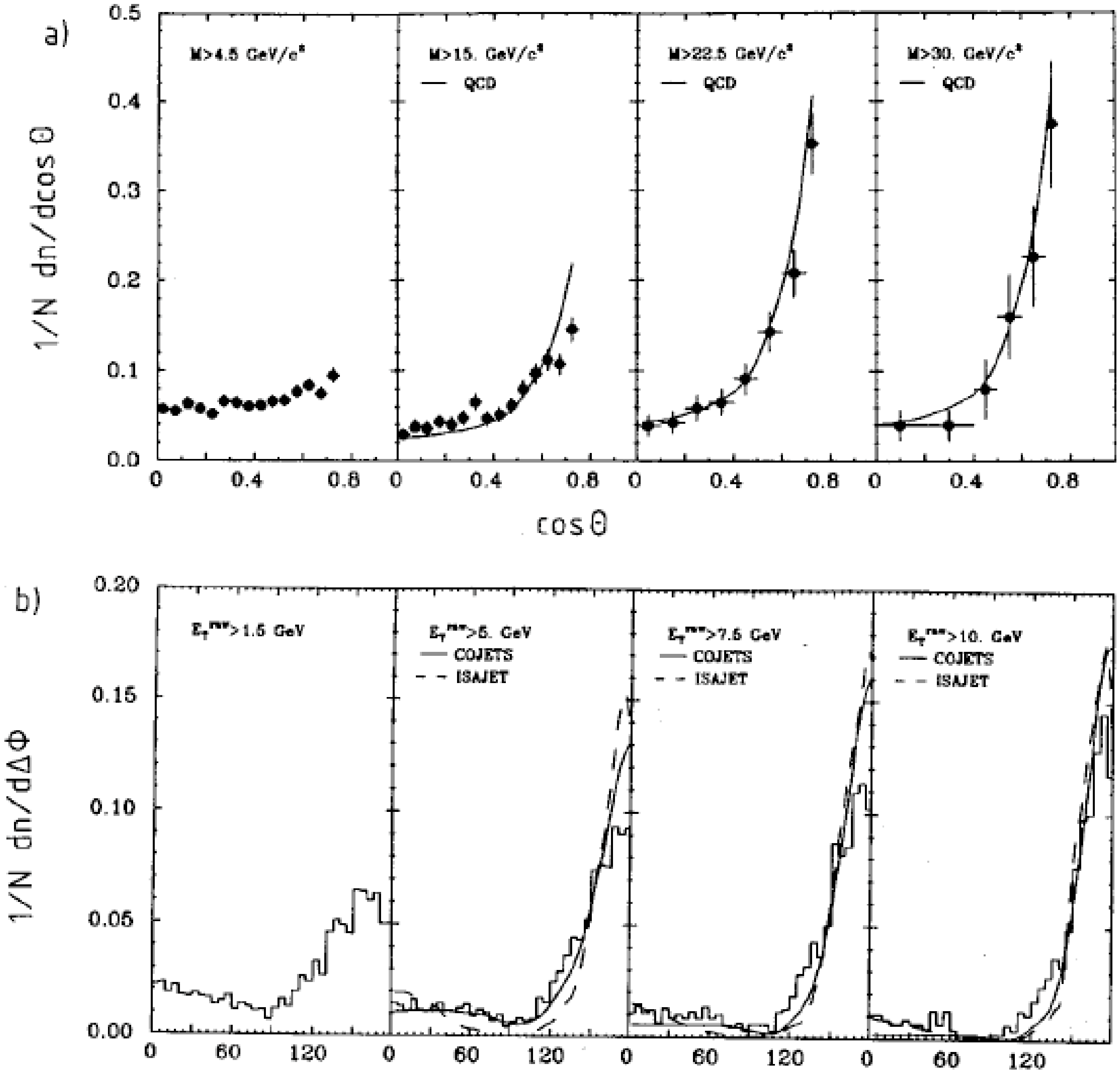
UA1 minijets



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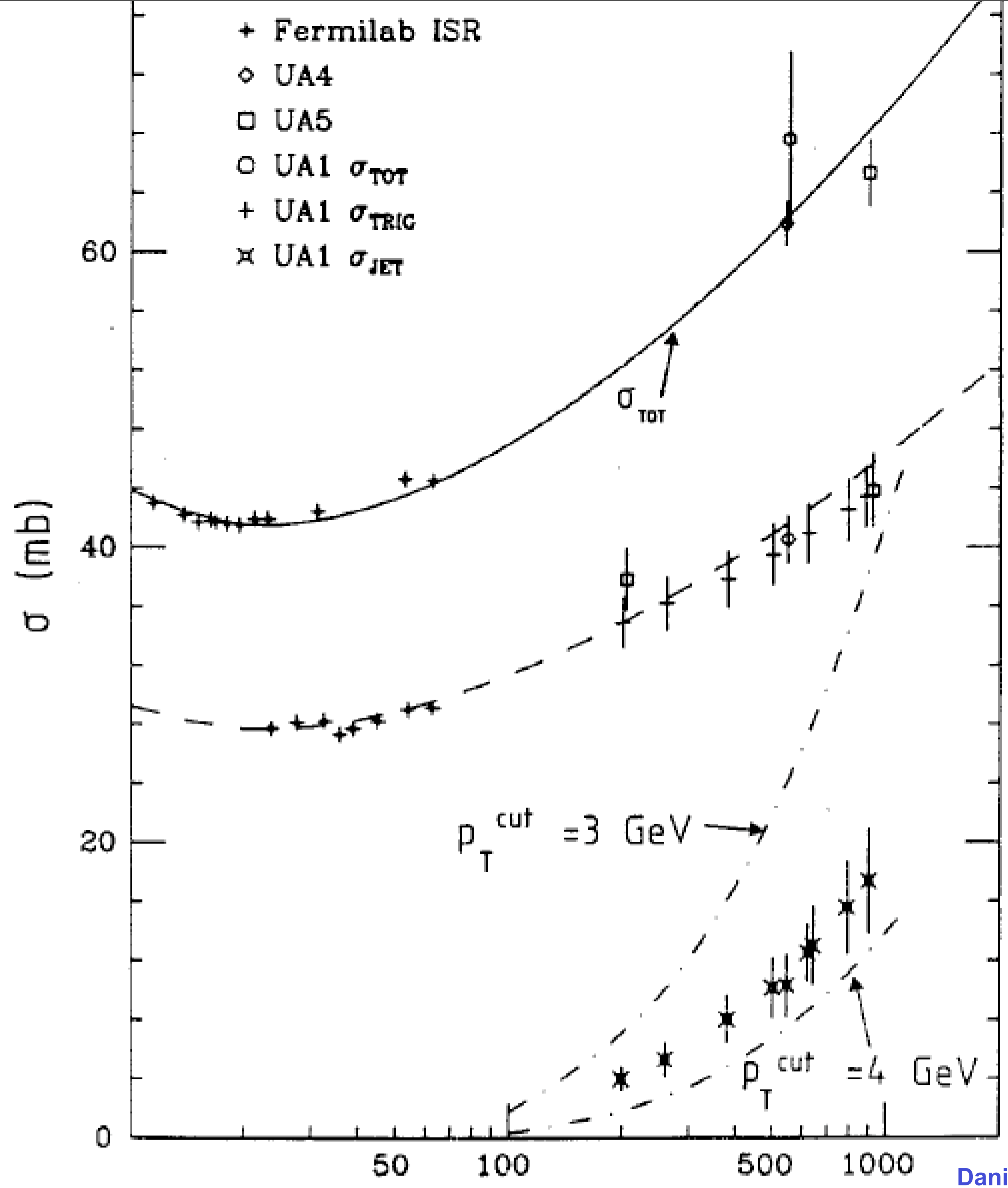

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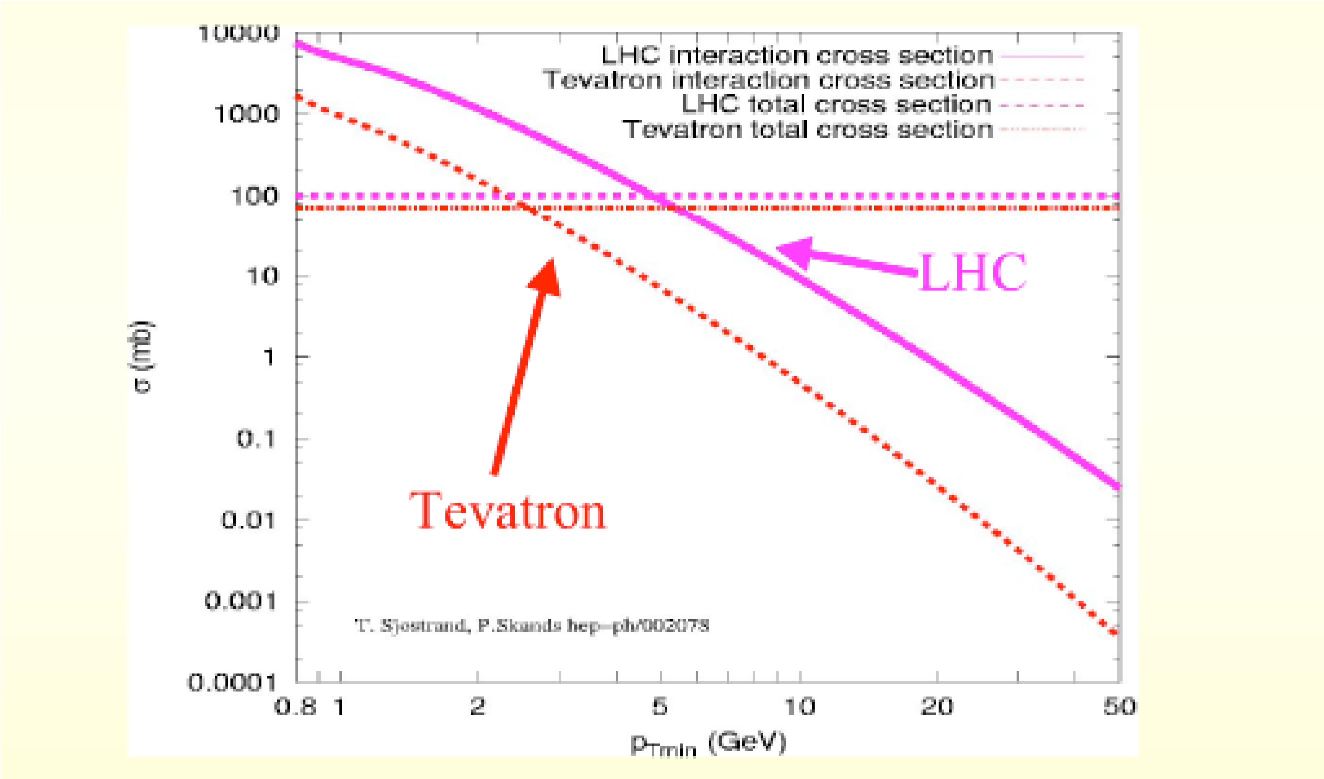
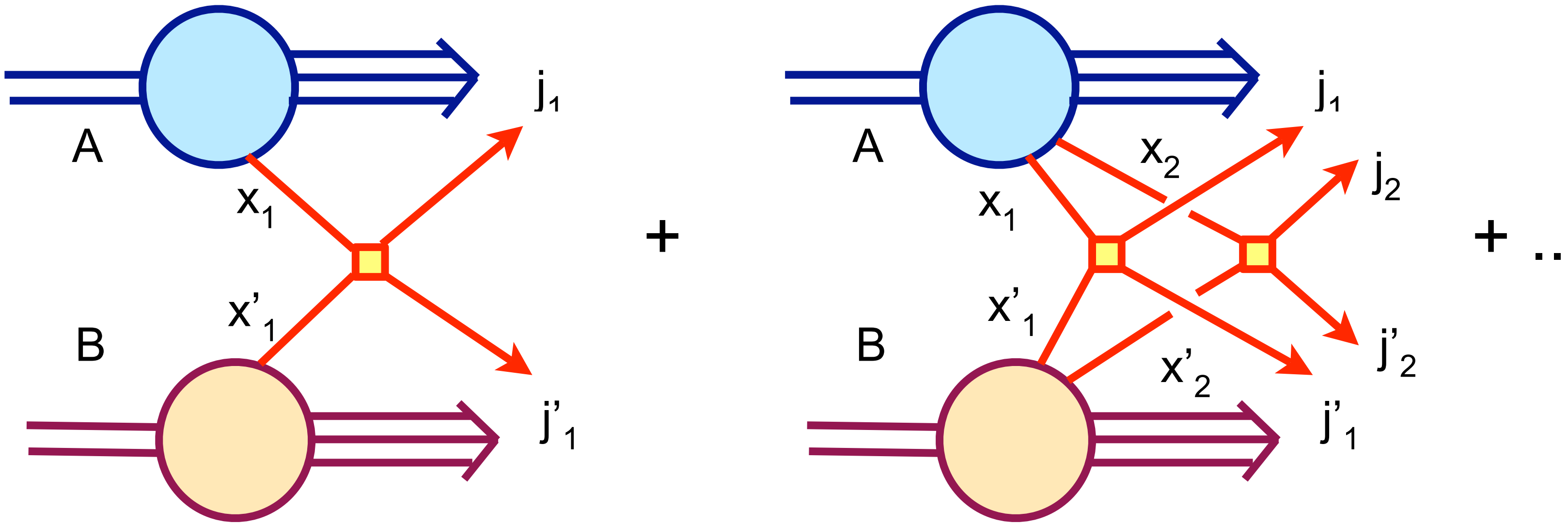


At high energy the integrated inclusive cross section of large pt jet production exceeds the value of the total cross section
Unitarity is restored by introducing multiple parton interactions



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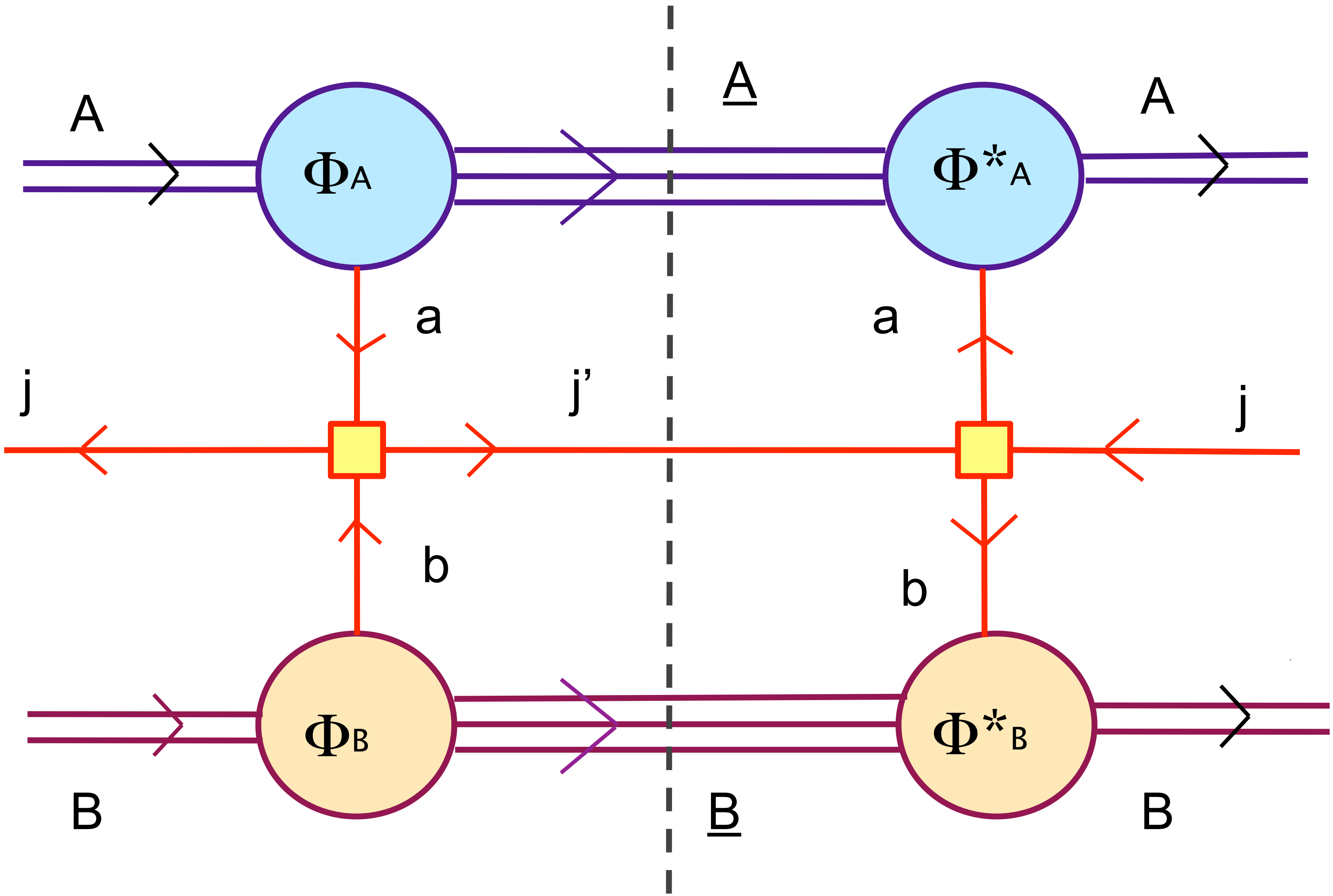
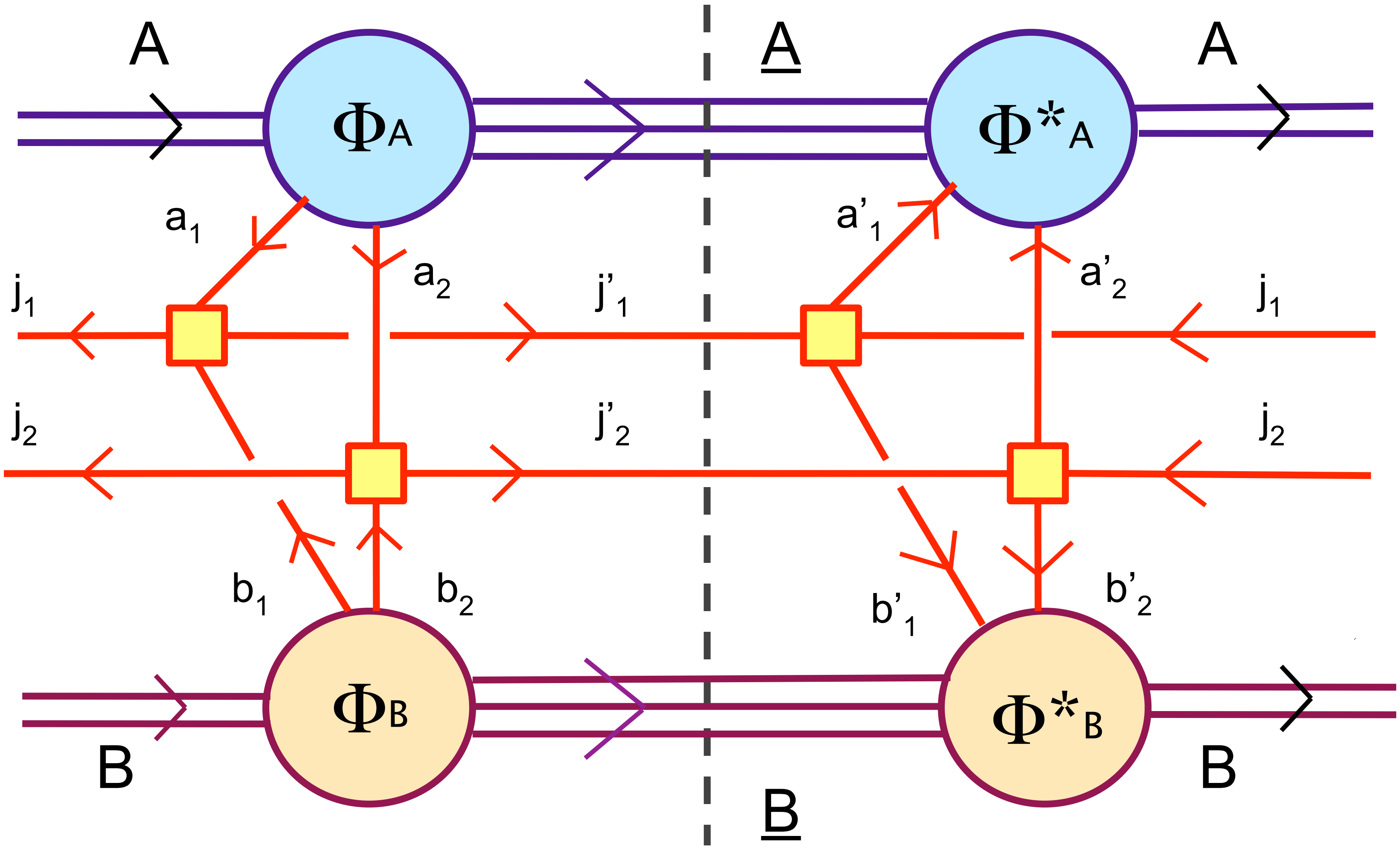
The ***single parton scattering cross section section*** is given by the following discontinuity



The ***double parton scattering cross section section*** is given by the discontinuity of the following diagram



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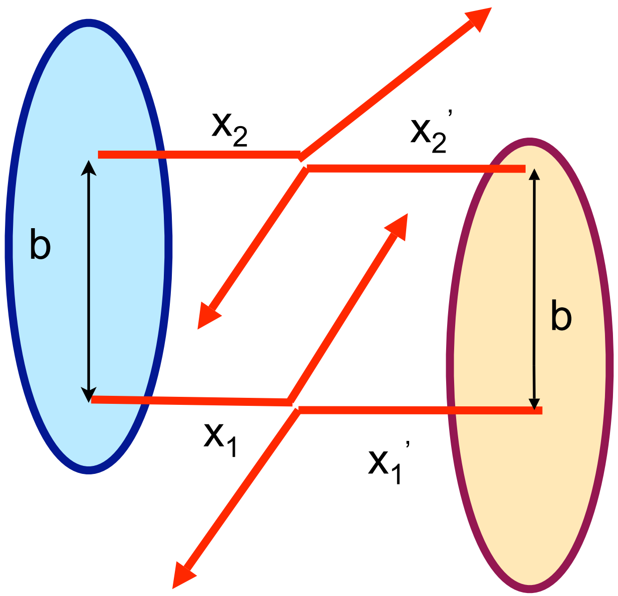
The ***basic assumption*** is that the scale of the ***transverse momenta*** and of the ***virtualities*** of the parton lines attached to the non perturbative vertices is ***not growing with s***

The resulting ***picture of the interaction*** is ***geometrical***



The two parton interactions are localized in two regions, with a size of order $1/p_t$, much smaller with respect to the overlap region of the matter distribution of the two hadrons. The relative transverse distance between the two pairs of interacting partons needs hence to be the same in the two hadrons, in order to have the necessary alignment for the interaction to occurr

The inclusive double parton-scattering cross-section, for two parton processes *A* and *B* in a *pp* collision, is hence given by

$$\sigma^D_{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int D_{ij}(x_1, x_2; b)\hat{\sigma}^A_{ik}(x_1, x_1')\hat{\sigma}^B_{jl}(x_2, x_2')D_{kl}(x_1', x_2'; b)dx_1 dx_1' dx_2 dx_2' d^2b$$

where $D_{ij}(x_1, x_2, b)$ are the double parton distribution functions

Notice that, as a consequence of the dependence on the relative transverse distance *b*, the ***double parton distributions*** are ***dimensional quantities, independent of the one body parton distributions usually considered in large p_t processes.*** The double parton distributions depend in fact explicitly on ***parton correlations***.

The expression of the cross section simplifies considerably after neglecting correlations in the momentum fraction *x*. In such a case the two-body parton distributions are given by

$$D_{ij}(x_1, x_2; b) = D_i(x_1)D_j(x_2)F_i^j(b)$$

The double parton scattering cross section is hence expressed in a simple way as a combination of products of single scattering cross sections

$$\sigma^D_{(A,B)} = \frac{m}{2} \sum_{ijkl} \Theta^{ij}_{kl} \sigma^S_{ij}(A) \sigma^S_{kl}(B)$$

where

$$\Theta^{ij}_{kl} = \int d^2b \, F^i_k(b) \, F^j_l(b)$$

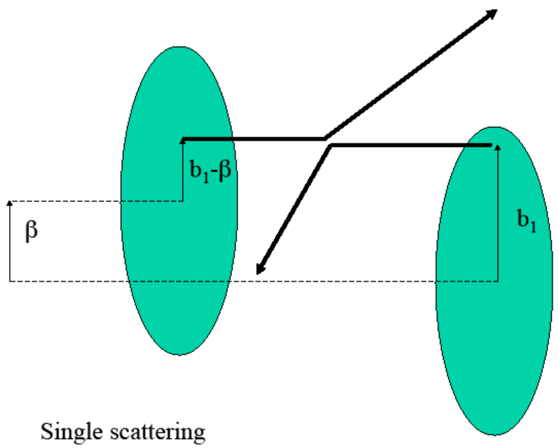
are geometrical coefficients with dimension an inverse cross section and ***depending directly on the transverse correlation of the different kinds of parton pairs in the hadron structure***

In the simplest case the functions F_k^i do not depend on the indices *i, k* and the coefficients are all equal to a universal constant. The double parton scattering cross section is hence given by

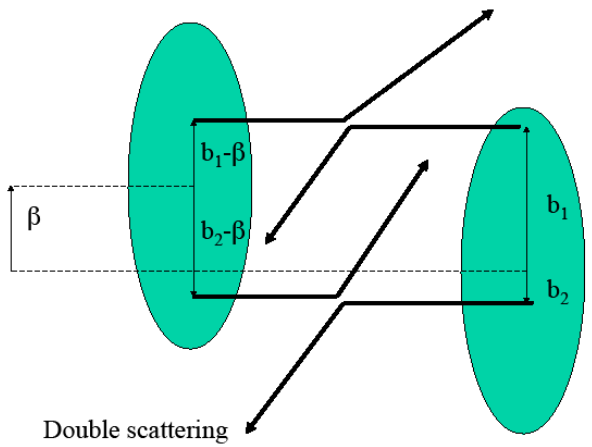
$$\sigma_D = \frac{1}{2} \frac{\sigma_S^2}{\sigma_{eff}}$$

The unitarity issue

Neglecting all longitudinal correlations and assuming independence on the flavor indices, one may easily write the *N*-parton inclusive cross section σ_N



$$\sigma_S = \int_{p_t^c} D(x) f(b) \hat{\sigma}(x,x') D(x') f(b-\beta) d^2b d^2\beta dx dx'$$



$$\begin{aligned}\sigma_D &= \frac{1}{2!}\int_{p_t^c} D(x_1) f(b_1) \hat{\sigma}(x_1,x_1') D(x_1') f(b_1-\beta) d^2b_1 dx_1 dx_1' \times \\ &\quad \times D(x_2) f(b_2) \hat{\sigma}(x_2,x_2') D(x_2') f(b_2-\beta) d^2b_2 dx_2 dx_2' d^2\beta \\ &= \int \frac{1}{2!}\Big(\int_{p_t^c} D(x) f(b) \hat{\sigma}(x,x') D(x') f(b-\beta) d^2b dx dx'\Big)^2 d^2\beta\end{aligned}$$

$$\sigma_N = \int \frac{1}{N!}\Big(\int_{p_t^c} D(x) f(b) \hat{\sigma}(x,x') D(x') f(b-\beta) d^2b dx dx'\Big)^N d^2\beta$$

f(b) is the density of partons in transverse space and $F(b) = \int d^2b' f(b-b') f(b')$

The integrand $\frac{1}{N!}\left(\int_{p_t^c} D(x)f(b)\hat\sigma(x,x')D(x')f(b-\beta)d^2bdxdx'\right)^N$ is dimensionless

and ***after normalization*** it may be understood as the probability to have *N* parton collisions:

$$\frac{(\sigma_S F(\beta))^N}{N!}e^{-\sigma_S F(\beta)} = \mathrm{P}_N(\beta) \qquad \sigma_S F(\beta) = \int_{p_t^c} D(x)f(b)\hat\sigma(x,x')D(x')f(b-\beta)d^2bdxdx'$$

One may hence express the hard cross section σ_H (namely the contribution to σ_{inel} of all events with ***at least*** one parton collision) as

$$\sigma_H = \sum_{N=1}^{\infty}\int d^2\beta \frac{(\sigma_S F(\beta))^N}{N!}e^{-\sigma_S F(\beta)} = \int d^2\beta\left[1-e^{-\sigma_S F(\beta)}\right]$$

self-shadowing cross section

While σ_S is divergent when p_t^c goes to zero, σ_H and all contributions to σ_H with a given number *N* of parton collisions are, on the contrary, finite when p_t^c goes to zero

Differently form σ_S and σ_D, σ_H is always smaller that σ_{in} and one may write

$$\sigma_{in} = \sigma_{soft} + \sigma_H$$

The ***single*** and the ***double parton inclusive cross sections*** are given by the ***average*** and by the ***second moment of the distribution in the number of collisions***

$$\langle N\rangle\sigma_H=\int d^2\beta\sum_{N=1}^{\infty}\frac{N\big[\sigma_SF(\beta)\big]^N}{N!}e^{-\sigma_SF(\beta)}=\int d^2\beta\sigma_SF(\beta)=\sigma_S$$

$$\begin{aligned}\frac{\langle N(N-1)\rangle}{2}\sigma_H &= \frac{1}{2}\int d^2\beta\sum_{N=2}^{\infty}\frac{N(N-1)\big[\sigma_SF(\beta)\big]^N}{N!}e^{-\sigma_SF(\beta)}\\ &= \frac{1}{2}\int d^2\beta\big[\sigma_SF(\beta)\big]^2=\sigma_D\end{aligned}$$

The relations

$$\langle N\rangle\sigma_H=\sigma_S \quad \text{and} \quad \frac{1}{2}\langle N(N-1)\rangle\sigma_H=\sigma_D$$

are ***not specific of the Poissonian case*** and can be derived on much more general grounds

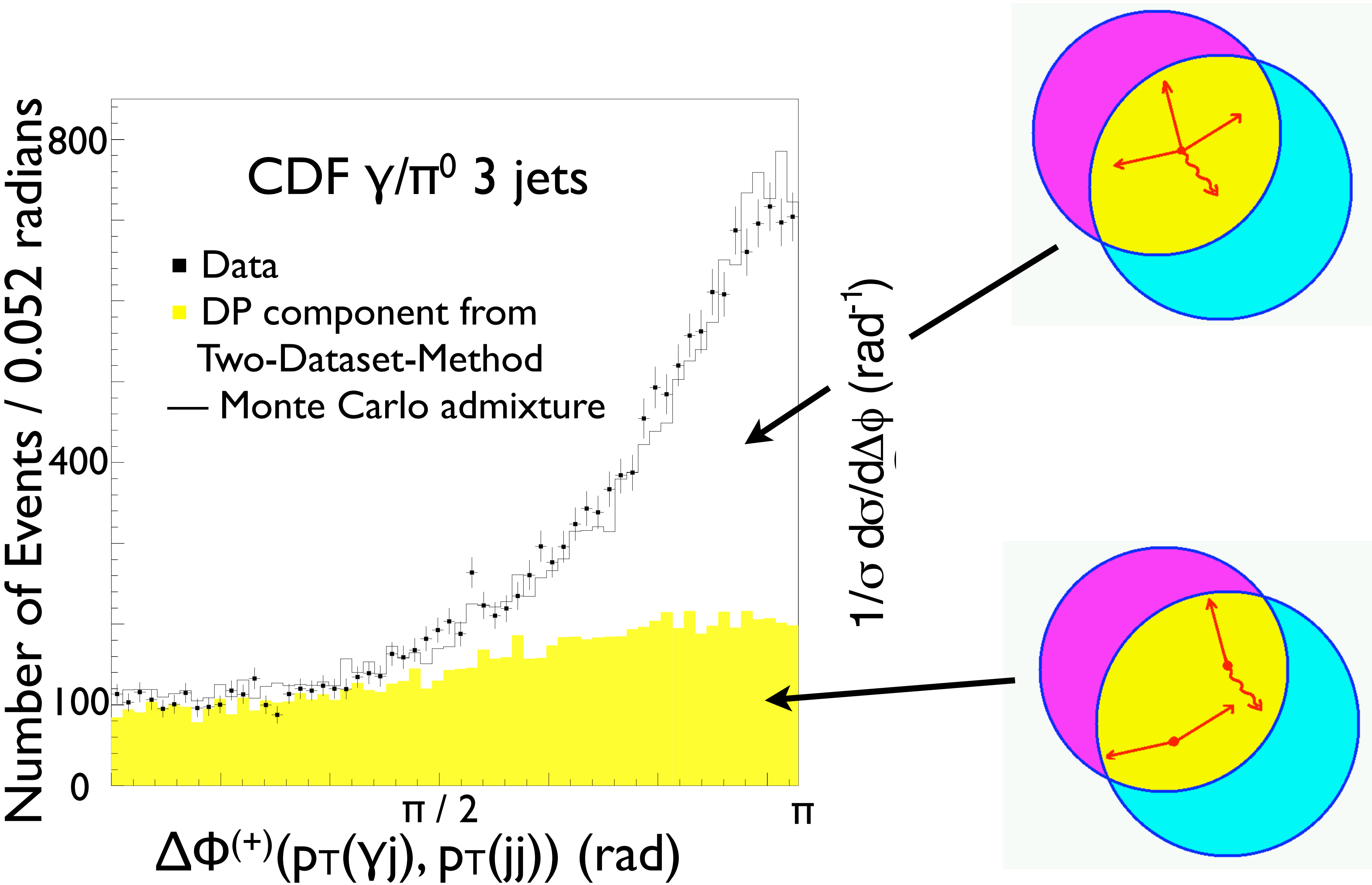
The same relations can in fact be obtained also when considering the most general case of multiparton distributions, namely **including all possible multi-parton correlations** (in particular the correlations induced by conservation laws).
On the other hand, the direct link, between inclusive cross sections and moments of the distribution in multiplicity of collisions, gets spoiled when taking into account also connected multiparton interactions, namely 3->3 etc. parton collision processes which, nevertheless, should not give rise to major effects in pp collisions even at LHC energies.

Rather generally, **the effective cross section hence represents a measure of the dispersion** of the distribution in the number of collisions. A small effective cross section corresonds to a large value of the dispersion.

$$\langle N(N-1)\rangle = \langle N\rangle^2 \frac{\sigma_{hard}}{\sigma_{eff}}$$

Notice that the Poissonian model is the basis off all present day implementations of multiple parton collisions in hadronic Montecarlos

Double parton scattering and the CDF result



The experimental indication of CDF is $\sigma_{eff} \simeq 11 \quad \text{mb}$

$$\sigma_D = \frac{1}{2}\frac{\sigma_S^2}{\sigma_{eff}}$$

The expectation of the simplest Poissonian model is however very different

parton density	effective cross section			
rms radius		R=.6 fm	R=.7 fm	R=.86 fm
$e^{-\frac{3}{2}\left(\frac{r}{R}\right)^2}$	$= \frac{8}{3}\pi R^2$	30 mb	41 mb	62 mb
$e^{-\sqrt{12}\left(\frac{r}{R}\right)}$	$\simeq \frac{71}{12}R^2$	21 mb	29 mb	44 mb

rms charge radius

here R is the rooth mean square hadron radius

One may hence conclude that **correlations between partons play an important role in the hadron structure**

Hadronic fluctuations

A source of correlations is the fluctuation of the hadron in different configurations, which is a phenomenon directly related to **hadron diffraction**

Hadron fluctuations may be implemented in a **multichannel eikonal model** of high energy hadronic interactions. As a result the approach gives a rather natural **generalization of the Poissonian model** of the hard cross section.

In the multichannel eikonal model the hadron state ψ_h is represented as a superposition of the eigenstates ϕ_i of the T matrix and the cross sections are combinations of the various cross sections σ_{ij}, which describe the interaction between the eigenstates ϕ_i and ϕ_j

$$\psi_h = \sum_i \alpha_i \phi_i$$

$$\begin{aligned} \sigma_{tot} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{tot}^{ij} \\ \sigma_{el} + \sigma_{sd} + \sigma_{dd} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{el}^{ij} \\ \sigma_{in} &= \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma_{in}^{ij} \end{aligned}$$

The hard cross section is analogously expressed as

$$\sigma_{hard} = \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \sigma^{ij}_{hard} = \sum_{i,j} |\alpha_i|^2 |\alpha_j|^2 \int d^2\beta \Big[1 - e^{-\sigma^{ij}_S(\beta)}\Big]$$

$$= \sum_{i,j,N} |\alpha_i|^2 |\alpha_j|^2 \int d^2\beta \frac{\left(\sigma^{ij}_S(\beta)\right)^N}{N!} e^{-\sigma^{ij}_S(\beta)}$$

which represents the obvious **generalization** of the expression obtained in the **Poissonian model**. The **increased dispersion**, with the consequent **decrease** of the value **of the effective cross section** as compared with the value obtained in the Poissonian model, is a natural feature of the multichannels.

For a qualitative understanding of the effect let us consider a simplest two channel toy model (**thanks to Mark Strikman**)

Let us assume that the hadron may appear with equal weights in two different configurations :

$$\psi_h = \frac{1}{\sqrt{2}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2$$

The hard cross section σ_{hard} is hence given by

$$\sigma_{hard} = \frac{1}{4}\sigma_{hard}^{11} + \frac{1}{2}\sigma_{hard}^{12} + \frac{1}{4}\sigma_{hard}^{22}$$

and the expression of the *N*-parton scattering inclusive cross section σ_N is

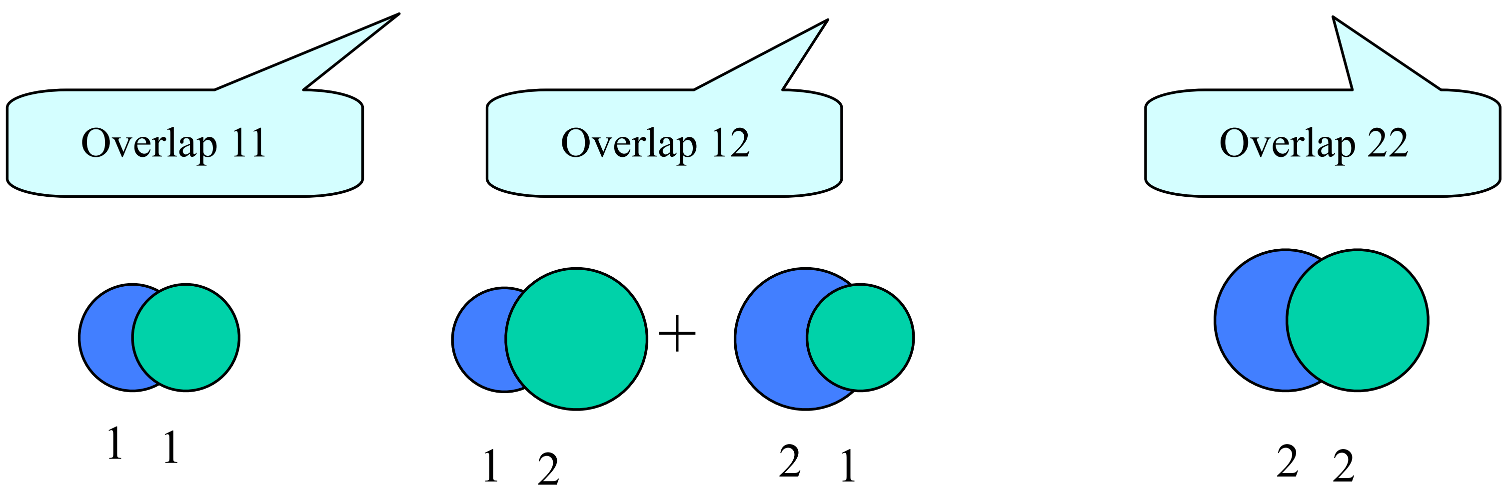
$$\sigma_N = \frac{\sigma_S^N}{N!}\left\{\frac{1}{4}\int \left[F_{11}(\beta)\right]^N d^2\beta + \frac{1}{2}\int \left[F_{12}(\beta)\right]^N d^2\beta + \frac{1}{4}\int \left[F_{22}(\beta)\right]^N d^2\beta\right\}$$

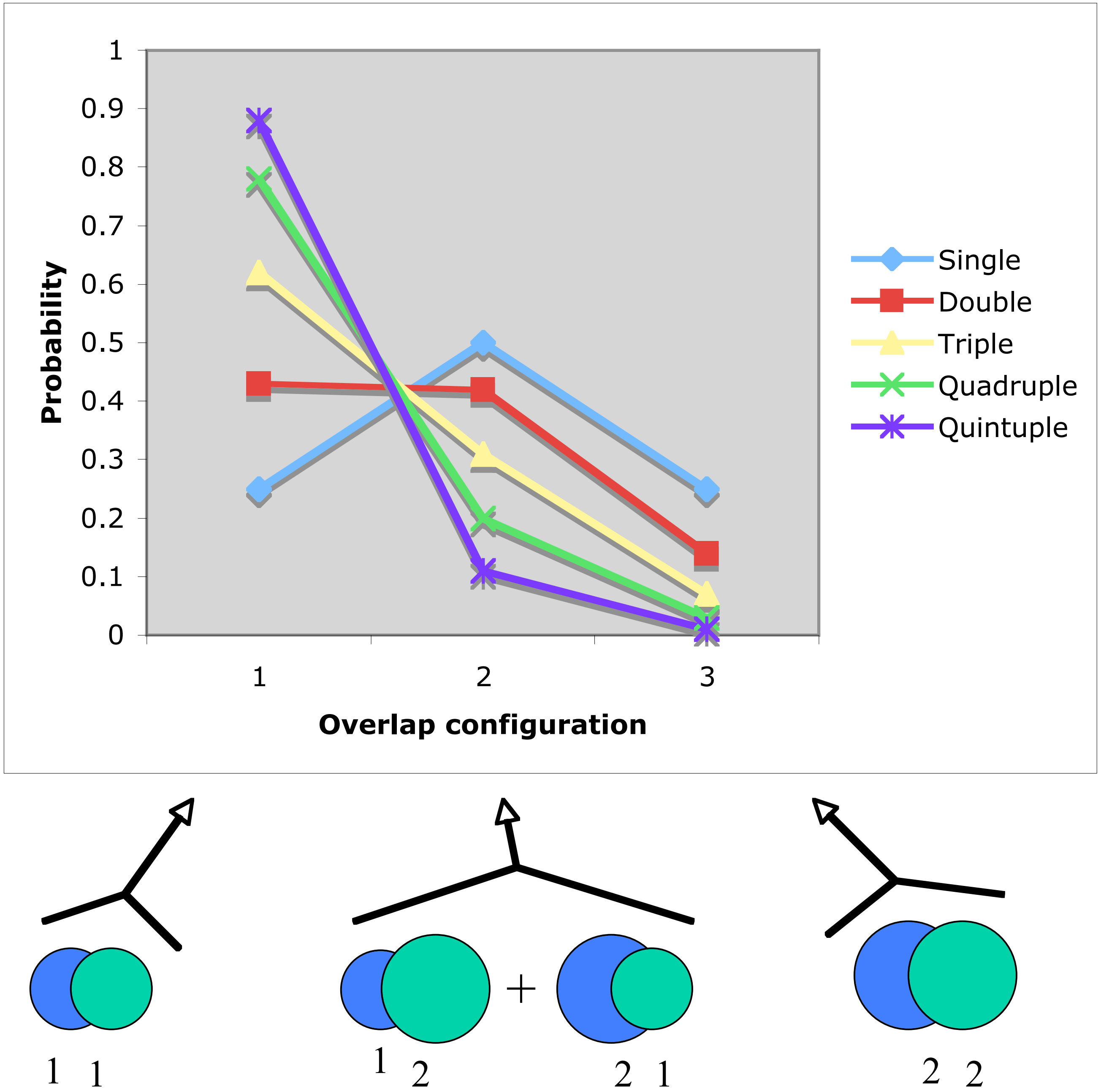
A simplest possibility to evaluate $\int [F_{ij}(\beta)]^N d^2\beta$ is to assume a Gaussian shape

$$F_{ij}(\beta) = \frac{1}{\pi(R_i^2 + R_j^2)} \times \exp\Big(\frac{-\beta^2}{R_i^2 + R_j^2}\Big)$$

while for the sizes of the two different configurations one may take $(R_1)^2 = R^2/2$ and $(R_2)^2 = 3R^2/2$, in such a way that the average hadron size is R^2.
The inclusive cross section σ_N is hence given by:

$$\sigma_N = \frac{\sigma_S^N}{N N!(\pi R^2)^{N-1}} \left\{ \frac{1}{4}\left(\frac{1}{\frac{1}{2}+\frac{1}{2}}\right)^{N-1} + \frac{1}{2}\left(\frac{1}{\frac{1}{2}+\frac{3}{2}}\right)^{N-1} + \frac{1}{4}\left(\frac{1}{\frac{3}{2}+\frac{3}{2}}\right)^{N-1} \right\}$$





Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
Overlap configuration
Single
Double
Triple
Quadruple
Quintuple
1 1
1 2
+
2 1
2 2

A simplest **eikonal model**, where all hadronic diffractive states are included in a single channel and which is capable of reproducing the total, elastic, single and double diffraction cross sections in high energy hadronic interactions is due to **Gotsman, Levin and Maor**

Utilizing the weights of the different channels, fitted to reproduce the total, elastic, single and double diffractive cross sections, one may estimate the value of the effective cross section at TeVatron and at the LHC

$$\sqrt(s) = 14\mathrm{TeV} \quad \sigma_{tot} = 114\mathrm{mb} \quad \sigma_{inel} = 71\mathrm{mb} \quad \sigma_{eff} = 12\mathrm{mb}$$

$$\sqrt(s) = 1.8\mathrm{TeV} \quad \sigma_{tot} = 81\mathrm{mb} \quad \sigma_{inel} = 50\mathrm{mb} \quad \sigma_{eff} = 10\mathrm{mb}$$

which is rather close to the experimental indication (at TeVatron about 11 mb)

Concluding summary

Multiple parton scatterings are one of the main features of high energy hadron interactions

The non perturbative input of multiple parton scatterings are the multi-parton distributions. Measuring multiple parton scatterings one may hence obtain information on the multiparton structure of the hadron.

The present experimental indication by CDF on ***double parton scatterings*** is that correlations between partons represent an important feature of the hadron struture

The small value of the effective cross section measured by CDF is an indication that ***fluctuations of the hadron structure are larger than naively expected***.

Some indication on the fluctuations of the hadron structure are given by diffraction. Configurations with different sizes interact with different strengths in such a way that multiple parton interactions are more likely when the hadron fluctuates in a compact configuration